FILTRATION THROUGH A POROUS BARRIER

BETWEEN TWO COMMUNICATING VESSELS

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In the practice of laboratory experimental work directed towards studying the properties of porous materials there is extensive use of studying nonsteady state filtration regimes for compressed fluids [1-4] which make it possible not only to determine the filtration parameters of porous materials, but also to evaluate the applicability of generally accepted filtration models [5, 6] for nonsteady-state flow. However, the existence of procedures for treating data are based on a quasistatic approximation, i.e., on the assumption of a slow change in pressure gradient.

In the present work the problem is considered of rapid gas or liquid flow from one vessel of fixed volume to another through a specimen of porous material. An equation has been obtained connecting characteristic relaxation time in the system with specimen permeability.

Let there be two communicating vessels of volume V_0 and V_1 separated by a porous barrier of cylindrical shape with length L and cross sectional diameter d. We assume that the system is filled with gas or liquid which flows isothermally from one vessel to the other through the barrier. The equation of state is taken in the form

$$p = c^2(\rho - \rho_a) + p_a, \tag{1}$$

where ρ and p are density and pressure; c is isothermal sound velocity; ρ_a and p_a are constants. Equation (1) describes an ideal gas when $\rho_a = 0$ and $p_a = 0$, and the majority of real liquids if the range of change in density is small, i.e., $|\rho - \rho_a|/\rho_a \ll 1$.

We direct axis 0x in space so that the porous barrier is projected on section [0, L], and the positive direction relates to the direction from the vessel with V_0 to vessel with V_1. Flow will be assumed to be unidimensional and to satisfy the Darcy rule [5, 6]

$$u = -\frac{k}{\mu}\frac{\partial p}{\partial x}, \tag{2}$$

where k and μ are permeability and viscosity which are assumed to be constant.

During filtration a continuity equation is satisfied

$$\partial m\rho/\partial t = -\partial \rho u/\partial x \tag{3}$$

(m is porosity, which is also assumed to be constant).

From (1)-(3) an equation follows for function $\rho = \rho(t, x)$:

$$\frac{\partial\rho}{\partial t} = \frac{kc^2}{m\mu}\frac{\partial}{\partial x}\rho\frac{\partial}{\partial x}\rho. \tag{4}$$

Functions $\rho(t, 0)$ and $\rho(t, L)$ give the value of density in the corresponding vessels. In view of the law of mass conservation there are boundary conditions

$$\begin{gathered} V_0\rho(t, 0) = V_0\rho(0, 0) - S\int_0^t (\rho u)(s, +0)\,ds, \\ V_1\rho(t, L) = V_1\rho(0, L) + S\int_0^t (\rho u)(s, L-0)\,ds, \quad S = \frac{\pi d^2}{4}. \end{gathered} \tag{5}$$

From (1), (2), (5) it emerges that

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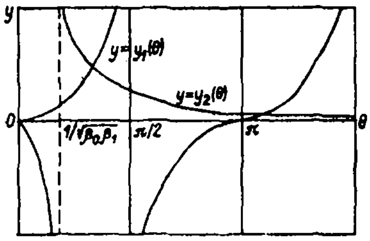

Fig. 1

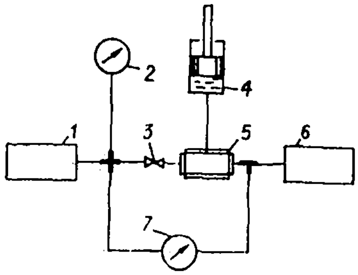

Fig. 2

$$V_0\frac{\partial\rho}{\partial t}(t,\,0)=\frac{Skc^2}{\mu}\left(\rho\frac{\partial\rho}{\partial x}\right)(t,\,+0),$$
$$V_1\frac{\partial\rho}{\partial t}(t,\,L)=-\frac{Skc^2}{\mu}\left(\rho\frac{\partial\rho}{\partial x}\right)(t,\,L-0). \tag{6}$$

It is assumed that in the initial instant at the boundary of one of the vessels and the porous barrier a jump in density is created $\rho(0,\,0)=\rho_0$, $\rho(0,\,x)=\rho_1, x\in(0,L]$. It is noted that these initial conditions assume a special selection of functional space in which the solution of the problem is sought.

With the passage of time density in the system varies and it tends towards constant ρ_∞, which may be determined from the law of mass conservation $\rho_\infty=(V_0\rho_0+V_1\rho_1+mLS\rho_1)/(V_0+V_1+$ mLS).

We consider the case where deviations of density from the limiting value are small: $|\rho-\rho_\infty|/\rho_\infty\ll 1$. The problem (4), (6) is converted into a linear problem

$$\frac{\partial\rho}{\partial t}=\frac{kc^2\rho_\infty}{m\mu}\frac{\partial^2\rho}{\partial x^2},\quad V_0\frac{\mu}{Skc^2\rho_\infty}\frac{\partial\rho}{\partial t}(t,\,0)=\frac{\partial\rho}{\partial x}(t,\,+0),$$
$$V_1\frac{\mu}{Skc^2\rho_\infty}\frac{\partial\rho}{\partial t}(t,\,L)=-\frac{\partial\rho}{\partial x}(t,\,L-0).$$

We change over to dimensionless values: $x=L\xi$, $t=m\mu L^2\tau/(kc^2\rho_\infty)$, $\rho=\rho_\infty v$, $\rho_0=\rho_\infty v_0$, $\rho_1=\rho_\infty v_1$, $\beta_0=V_0/(mSL)$, $\beta_1=V_1/(mSL)$.

In the dimensionless state the problem has the form

$$\frac{\partial v}{\partial\tau}=\frac{\partial^2 v}{\partial\xi^2},\quad \beta_0\frac{\partial v}{\partial\tau}(\tau,\,0)=\frac{\partial v}{\partial\xi}(\tau,\,+0),\quad \beta_1\frac{\partial v}{\partial\tau}(\tau,\,1)=-\frac{\partial v}{\partial\xi}(\tau,\,1-0); \tag{7}$$

$$v(0,\,\xi)=\begin{cases}v_0, & \xi=0,\\ v_1, & 0<\xi\leqslant 1.\end{cases} \tag{8}$$

Now we write the functional space in which problem (7), (8) will be solved. For this purpose we introduce into the space of continuous complex-valued functions $C_C[0,\,1]$ a scalar product

$$(\varphi_1,\,\varphi_2)=\beta_0\varphi_1^*(0)\,\varphi_2(0)+\beta_1\varphi_1^*(1)\,\varphi_2(1)+\int_0^1\varphi_1^*(\xi)\,\varphi_2(\xi)\,d\xi.$$

Supplement $C_C[0,\,1]$ according to the corresponding norm is a Hilbert space G. We determine in $C_C^2[0,\,1]$ operator A by the equations

$$(Af)(\xi)=\begin{cases}\left.\dfrac{1}{\beta_0}\dfrac{df}{d\xi}\right|_{\xi=0}, & \xi=0,\\ \left.-\dfrac{1}{\beta_1}\dfrac{df}{d\xi}\right|_{\xi=1}, & \xi=1,\\ \dfrac{d^2f}{d\xi^2}(\xi), & \xi\in(0,\,1)\end{cases}$$

(A is a symmetrical operator in G which permits closure [7, 8]). We shall indicate closing of operator A by the same symbol.

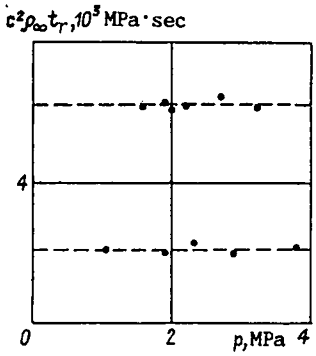

Fig. 3

In view of the substance of operator A its defect indices are equal and therefore A is expanded to a self-conjugating operator. Self-conjugated expansion is also indicated in terms of A, and then D_A is the region for determining A.

Operator A has a discrete spectrum λ_n $(n = 0, 1, \ldots)$, $\lambda_0 = 0 > \lambda_1 > \ldots > \lambda_n > \lambda_{n+1} > \ldots$, $\lambda_n \to -\infty$ with $n \to +\infty$. In fact for any $f \in D$

$$(f, Af) = -\int_0^1 \left|\frac{df}{d\xi}\right|^2 d\xi \leqslant 0,$$

and therefore the spectrum of A lies on a beam $(-\infty, 0]$. Since $Af = 0$ with $f = \text{const}$, then $\lambda_0 = 0$ is the eigenvalue of operator A. Furthermore, let $\lambda = -\theta^2$, $\theta > 0$. We consider the equation $Af - \lambda f = g$ $(f \in D_A,\ g \in G)$, and it is broken down into

$$\left(\frac{d^2}{d\xi^2} + \theta^2\right) f(\xi) = g(\xi), \quad \xi \in (0, 1); \tag{9}$$

$$\left(\frac{1}{\beta_0}\frac{d}{d\xi} + \theta^2\right) f(0) = g(0), \quad \left(-\frac{1}{\beta_1}\frac{d}{d\xi} + \theta^2\right) f(1) = g(1). \tag{10}$$

From (9) it follows that with $\xi \in [0, 1]$

$$f(\xi) = \frac{1}{\theta}\int_0^1 \sin\theta(\xi - \eta)\, g(\eta)\, d\eta + a \sin\theta\xi + b\cos\theta\xi, \tag{11}$$

where a and b do not depend on ξ. We substitute (11) in (10), then

$$\begin{gathered}\frac{1}{\beta_0}\theta a + \theta^2 b = g(0),\\ \theta\left(-\frac{1}{\beta_1}\cos\theta + \theta\sin\theta\right) a + \theta\left(\frac{1}{\beta_1}\sin\theta + \theta\cos\theta\right) b = g(1) + \int_0^1 \left(\frac{1}{\beta_1}\cos\theta(1-\eta) - \theta\sin\theta(1-\eta)\right) g(\eta)\, d\eta.\end{gathered} \tag{12}$$

If the determinant

$$\Delta(\theta) = \begin{vmatrix} \frac{1}{\beta_0} & \theta \\ \left(-\frac{\cos\theta}{\beta_1} + \theta\sin\theta\right) & \left(\frac{\sin\theta}{\beta_1} + \theta\cos\theta\right) \end{vmatrix} = \left(\frac{1}{\beta_0\beta_1} - \theta^2\right)\sin\theta + \theta\left(\frac{1}{\beta_0} + \frac{1}{\beta_1}\right)\cos\theta$$

differs from zero, then system (12) determines a and b as linear continuous functionals of g, and Eq. (11) is a linear continuous operator inverse to operator $(A - \lambda I)$. In this case λ pertains to the resolvent set of operator A. If

$$\Delta(\theta) = 0, \tag{13}$$

then $\lambda = -\theta^2$ is the eigenvalue of A. Equation (13) has a countable set of real solutions with a limiting point at infinity. Thus, confirmation of the spectrum of operator A is proven.

TABLE 1

Characteristics	Specimen 1	Specimen 2
Lithological type	Limestone	Sandstone
Porosity m, 10^{-2}	6,7	5,1
Length L, 10^{-2} m	5,07	3,74
Diameter d, 10^{-2} m	2,87	2,88
Permeability k, measured with a steady-state regime, 10^{-17} m^2	4,5	9,6
Permeability calculated by Eq. (17), 10^{-17} m^2	4,2	9,1

Let $f_n = f_n(\xi) = \alpha_n(\cos\theta_n\xi - \beta_0\theta_n\sin\theta_n\xi)/\sqrt{1+\beta_0^2\theta_n^2}$ $(n=0,1,\ldots)$ be eigenfunctions corresponding to the eigenvalues of operator A: $\lambda_n = -\theta_n^2$, $\theta_0 = 0$, $\theta_n < \theta_{n+1}$. Constants α_n are found from the normalization condition $(f_n, f_n) = 1$:

$$\alpha_0^{-2} = 1 + \beta_0 + \beta_1,\quad \alpha_n^{-2} = \frac{1}{2}\left(1 + \frac{\beta_0}{1+\beta_0^2\theta_n^2} + \frac{\beta_1}{1+\beta_1^2\theta_n^2}\right)\quad (n = 1, 2, \ldots).$$

Furthermore, we designate w an element of G prescribed by Eq. (8). Then problem (7), (8) is reduced to finding function $v = v(\tau)$ with values in G satisfying the equation $dv/d\tau = Av$ and the initial condition $v(0) = w$. This problem has a solution which may be presented in the form of a series

$$v = v(\tau) = e^{\tau A}w = \sum_{n=0}^{+\infty} e^{-\theta_n^2\tau}(f_n, w)f_n. \tag{14}$$

In this way

$$(f_0, w) = (\beta_0 v_0 + \beta_1 v_1 + v_1)\alpha_0,$$

$$(f_n, w) = (v_0 - v_1)\alpha_n\beta_0/\sqrt{1+\beta_0^2\theta_n^2}\quad (n = 1, 2, \ldots).$$

We consider in more detail the particular case when $\beta_0, \beta_1 \gg 1$, $\beta_0/\beta_1 \sim 1$. Equation (13) may be rewritten in the form

$$\tan\theta = -(\beta_0 + \beta_1)\theta/(1 - \beta_0\beta_1\theta^2), \tag{15}$$

the solution of which determines points of intersection of the curves $y = y_1(\theta) = \tan\theta$ and $y = y_2(\theta) = -(\beta_0 + \beta_1)\theta/(1 - \beta_0\beta_1\theta^2)$ (Fig. 1). It is easy to see that $\theta_n \gg \theta_1$ with $n > 1$, and with $\tau \gg 1/\theta_2^2$ the solution determines the first two terms in sum (14). From (15) we obtain

$$\theta_1 = (1/\beta_0 + 1/\beta_1)^{1/2} + O(1/\beta_0^{3/2}). \tag{16}$$

With $\tau \gg 1/\theta_2^2$ the value of density in the system approaches exponentially a constant value. In dimensional values according to (16) the relaxation time

$$t_r = LV_0V_1\mu/(c^2\rho_\infty k(V_0 + V_1)S). \tag{17}$$

With the aim of proving this theory a series of experiments was carried out. A diagram of the experimental device for studying nonsteady state filtration of fluids by the transfer method is shown in Fig. 2. A cylindrical specimen of porous material with a rubber ring is placed in core-holder 5 and it is squeezed by a system of a side hydraulic squeezer 4. Transfer of gas from the input vessel 1 to the output vessel 6 is initiated by opening valve 3. The process of transfer is monitored by a manometer 2 and a differential manometer 7.

Given in Fig. 3 are the results of experiments for proving Eq. (17). Experiments were carried out in two specimens with different average pressures and temperature T = 295°K. Methane gas was used as the working agent.

The inlet vessel of the device had a volume $V_0 = 4.32\cdot10^{-4}$ m^3, and for the outlet vessel $V_1 = 9.48\cdot10^{-4}$ m^3. Parameters for porous material specimens are given in Table 1. For the first specimen $\beta_0 = 198$, $\beta_1 = 434$, and for the second $\beta_0 = 348$, $\beta_1 = 764$. Thus, the assumptions adopted with respect to β_0 and β_1 are satisfied. All of the tests were carried out at pressures up to 4.0 MPa. In recording pressure value they did not differ from the limiting pressure by more than 10%. This made it possible to assume a viscosity value for methane in all tests of $\mu = 1.14 \cdot 10^{-5}$ Pa·sec (with a relative error not worse than ±0.03). Calculated permeability values for average values of the product $c^2\rho_\infty t_r$ are given in Table 1. They agree quite well with the results of steady-state measurements.

Thus, on the basis of the studies carried out it is possible to conclude that in the conditions considered the filtration process is well described by equations of an elastic regime. The method for determining filtration parameters according to typical relaxation time is new and it gives results conforming with those obtained in steady-state measurements.

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ACOUSTIC EFFECT ON THE HEAT-TRANSFER AND FLOW PARAMETERS OF A COMPOUND JET IN AN INCIDENT FLOW

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Processes involving the interaction of small perturbations (acoustic vibrations, vibrations of a surface with gas flows), of interest in both scientific investigations and in practical applications, are encountered in problems dealing with the transition of a laminar boundary layer to a turbulent boundary layer, the sensitivity of turbulent flows to acoustic vibrations, and the control of the aerodynamic and thermal characteristics of power plants [1-3].

Here, we examine the effect of acoustic vibrations on heat transfer and the hydrodynamic parameters in a compound jet discharged from a system of circular holes counter to a free-stream flow.

Tests were conducted in jets produced by an EDP-104A-50 electric-arc plasmatron, in an ohmic gas heater, and in the working section of a T-124 low-velocity low-turbulence wind tunnel.

The models (Fig. 1), made in the form of cylinders 1, were positioned with their end counter to an incoming flow of air 2. Air 4, formed into a compound jet, was fed through the internal volume of the models and seven circular holes in the end part 3. The dynamic

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